MATHS SL

Overall grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 15	16 – 31	32 – 45	46 – 57	58 – 69	70 – 82	83 – 100

Internal assessment

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 7	8 – 13	14 – 19	20 – 23	24 – 28	29 – 33	34 – 40

The range and suitability of the work submitted

The vast majority of schools selected tasks from the set of tasks offered by the IBO. In a few cases mathematics HL tasks were presented, or teachers had modified the IB tasks in some way. If the teacher did not review the task in the light of the SL criteria, this usually hindered candidates from achieving the highest levels of the assessment criteria. Teachers are reminded that HL tasks must be assessed against SL expectations, and that any modifications to tasks found elsewhere must be viewed in the light of the criteria so as to allow for maximum performance by candidates. It is also important that teachers submit solution keys for all tasks, whether or not taken from the set published by the IB. Teachers are required to submit solution keys as they assist greatly in the moderation process.

Many schools did not submit background information as to previous knowledge of the topics used in the task, or the availability and expectations of use of technology. In order for the moderators to be able to confirm teacher marks it is important that they understand the expectations of the classroom teachers.

Candidate performance against each criterion

Criterion A: There are still many cases where inappropriate notation is not being penalized. Use of calculator/computer notation will generally preclude a level 2. Candidates and teachers are lax in addressing issues of accuracy with proper notation. If answers are approximate then some form of approximately equals symbol is necessary. In modelling tasks candidates often use the same variable (usually ‘*y*’) for multiple model functions. This leads to confusion when comparisons are made and should be avoided. Candidates should be taught to use subscripted variables to distinguish between distinct models for the same behaviour.

Criterion B: Communication continues to improve, especially with graphs. Use of graphing software has allowed candidates to create clear and useful graphs that are well-labelled. Few candidates offered work in a question-and-answer format. Some candidates are producing work that is overly long (one piece was 130 pages!). Candidates should learn that a feature of good communication is the ability to present ideas in a concise manner.

Criterion C: In the Type I tasks candidates were quite successful in attaining level 4. However, most continue to have difficulty properly validating their conjectures. Often they simply substituted values of the variable (say *n*) into their proposed general statement and showed that the result matched the data they used to generate the statement in the first place. They must learn that they should be checking the results against the original pattern of behaviour and use additional values rather than those already found. Teachers are reminded that a **formal** explanation (e.g. an algebraic proof) is sufficient for level 5 (and level 5 for criterion D), and that no further testing is required. The exploration of scope and limitations is often brief and superficial. Candidates should be encouraged to consider all possibilities, including negatives, rationals, and irrationals, and to test these using the power of their graphic display calculators (GDC).

In the Type II tasks candidates are proving better at identifying variables and considering constraints. Identification of parameters often takes place as a part of the analysis, but their role should be made clear at some point. If the candidate chooses to develop a model function by using a series of graphical transformations then these should be seen. Comments such as “I tried a number of values for *k* and found this function fit best” are not sufficient and constitute only an attempt at analysis. When discussing the quality of fit many candidates offer superficial comments. While a quantitative analysis of fit is not required at SL, there should be some significant comments on how well the model matches the data. Applying the model function to a new set of data and commenting on the quality of fit is sufficient for level 5. (The necessary modifications to improve the fit are rewarded in criterion D).

A minority of candidates used regression analysis on a GDC or computer as the primary tool for model development. Teachers and candidates are reminded that this approach can achieve only a maximum of 2 since the requisite analytical steps are missing.

Criterion D: The most important aspects of criterion D are interpretation in context, and consideration of reasonableness and accuracy. Any model function simply approximates the real situation. Candidates should consider how good the model is and whether making it more accurate is reasonable. “How good does it have to be before it’s good enough?” is a question they should ask themselves. Many candidates get caught up in the mathematical analysis to a point where they forget what the problem is about. For example, comments about slopes or asymptotes should be interpreted in terms of rates and limiting behaviours within the context of the task. A purely mathematical interpretation of the results cannot achieve more than level 2.

Criterion E: Increased access to better technological resources has improved achievement for this criterion. However, the effective use of such resources continues to elude many candidates. The use of more graphs to show development, or to confirm a pattern of behaviour, is more effective than offering a single graph. The use of multiple graphs on the same axes to compare changes in parameters is more effective than a series of graphs on different axes. The use of printed output is not required, especially where access to technology is limited, but hand-drawn replications of graphs seen on the screen of a computer or GDC must be done carefully and accurately.

Criterion F: Most candidates achieved level 1. This recognizes that the candidate made a serious attempt to complete the task to the best of their ability. Level 0 should be awarded only in cases where it is clear that little attempt was made to complete the task, and the work is essentially unacceptable. Level 2 is reserved for work that has addressed all aspects of the task, and has demonstrated insight, precision, and significant understanding. The work should be truly admirable and not simply judged against the normal quality of the candidate's work.

Recommendations for the teaching of future candidates

Candidates should learn that the use of notation and terminology must be virtually flawless to obtain level 2 for criterion A. Proof-reading the work with appropriate advice from the teacher and referring to appropriate documents should ensure that the candidate is successful here. Communication must be that of a mathematical paper, not a set of homework questions. The work must read smoothly and not require reference to the task itself for clarification. In an investigation candidates should ensure that what they believe is the general statement is properly confirmed and validated, with a full exploration of scope and limitations on the variables. In a modelling task the analysis must be their own first, with the use of regression only for confirmation and comparison. The model must be interpreted in context and judged against what is reasonable and accurate. The use of technology must go beyond pushing buttons to create graphs or diagrams. These must work well within the presentation of the work, supporting and reinforcing the solutions.

Further comments

Teachers should ensure that all requisite forms are properly completed and submitted with the sample. Background information greatly assists in the moderation process, and solution keys are required for all tasks.

Overall candidates and teachers are doing well with the portfolio tasks. Some work seen was superlative and many schools submitted samples that indicate excellent organization and thorough knowledge and understanding of the assessment. Where schools had multiple teachers there was evidence of internal standardisation so as to ensure consistency of marking. The one missing element was that very few teachers offered tasks of their own. The IB encourages teachers to be risk-takers and to design interesting tasks that will engage their students.

External assessment

Paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 14	15 – 28	29 – 40	41 – 53	54 – 65	66 – 78	79 – 90

The areas of the programme and examination that appeared difficult for the candidates

- trigonometric values for angles such as π, 0, $\frac{3\pi}{2}$
- conditional probability and finding probabilities using a tree diagram
- integration of functions of the form $f(ax+b)$
- working with logarithms
- quadratic-type trigonometric equations
- transformations of functions
- "show that" and "hence" in the command terms
- basic computation and algebraic manipulation

The levels of knowledge, understanding and skill demonstrated.

The levels of understanding varied widely, and a large number of excellent scripts were seen in this session. Overall, the candidates seemed to be very well-prepared for this examination. Many candidates were able to gain at least partial marks on most questions.

The majority of candidates demonstrated good knowledge in questions involving:

- geometric sequences and series
- arc length and sector areas in circles
- working with vectors
- composite functions
- integration of basic polynomials

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

This question was very well done by the majority of candidates. There were some who used a value of *r* greater than one, with the most common error being $r = 2$. A handful of candidates struggled with the basic computation involved in part (c).

Question 2

Most candidates answered part (a) correctly, using the product rule to find the derivative, and earned full marks here. There were some who did not know to use the product rule, and of course did not find the correct derivative.

In part (b), many candidates substituted correctly into their derivatives, but then used incorrect values for $\sin \pi$ and $\cos \pi$, leading to the wrong gradient in their final answers.

Question 3

This question was very well done by the majority of candidates. Some candidates correctly substituted the values into the formulas, but failed to do the calculations and write their answers in finished form. Nearly all used the correct method of subtracting the sector areas in part (b), though multiplying with fractions proved challenging for some candidates.

Question 4

While nearly every candidate answered part (a) correctly, many had trouble with the other parts of this question.

In part (b), many candidates did not multiply along the branches of the tree diagram to find the required values, and many did not realize that there were two paths for P*(B)*. There were also many candidates who understood what the question required, but then did not know how to multiply fractions correctly, and these calculation errors led to an incorrect answer.

In part (c), most candidates attempted to use a formula for conditional probability found in the information booklet, but very few substituted the correct values.

Question 5

In part (a), most candidates successfully substituted using the double-angle formula for cosine. There were quite a few candidates who worked backward, starting with the required answer and manipulating the equation in various ways. As this was a "show that" question, working backward from the given answer is not a valid method.

In part (b), many candidates seemed to realize what was required by the word “hence”, though some had trouble factoring the quadratic-type equation. A few candidates were also successful using the quadratic formula. Some candidates got the wrong solution to the equation $\sin\theta = -1$, and there were a few who did not realize that the equation $\sin\theta = -\frac{3}{2}$ has no solution.

Question 6

While most candidates realized they needed to integrate in this question, many did so unsuccessfully. Many did not account for the coefficient of *x*, and failed to multiply by ½. Some of the candidates who substituted the initial condition into their integral were not able to solve for "*c*", either because of arithmetic errors or because they did not know the correct value for $\cos 0$.

Question 7

In part (a), nearly all candidates correctly substituted into the formula for the determinant. However, there were some candidates who made errors when simplifying this expression involving powers of e.

In part (b), most candidates realized they needed det ***A*** = 0. After this point, however, a multitude of different errors were seen. Among the candidates who realized they needed to use logarithms to solve the equation, most were unsuccessful in correctly applying the rules of logarithms. Many improper mathematical expressions such as "ln0" were seen in the working. Even the strongest candidates had trouble finding the final expression, with $a, b \in \mathbb{Z}$, and only a handful of candidates earned full marks on this question.

Question 8

The majority of candidates were successful on part (a), finding vectors between two points and using the scalar product to show two vectors to be perpendicular.

Although a large number of candidates answered part (b) correctly, there were many who had trouble with the vector equation of a line. Most notably, there were those who confused the position vector with the direction vector, and those who wrote their equation in an incorrect form.

In part (c), most candidates seemed to know what was required, though there were many who made algebraic errors when solving for the parameters. A few candidates worked backward, using $k = 1$, which is not allowed on a "show that" question.

In part (d), candidates attempted many different geometric and vector methods to find the area of the triangle. As the question said "hence", it was required that candidates should use answers from their previous working - i.e. $\mathrm{AC} \perp \mathrm{BD}$ and $\mathrm{P}(3,1)$. Some geometric approaches, while leading to the correct answer, did not use “hence” or lacked the required justification.

Question 9

Candidates showed good understanding of finding the composite function in part (a) and manipulating the quadratic in part (c), although there were some who did not seem to understand what the vector translation meant in part (b). There was more than one method to solve for *h* in part (d), and a pleasing number of candidates were successful in this part of the question.

Question 10

Part (a) seemed to be well-understood by many candidates, and most were able to earn at least partial marks here. Part (ai) was a "show that" question, and unfortunately there were some candidates who did not show how they arrived at the given expression.

In part (b), the concept seemed to be well-understood. Most candidates saw the necessity of using definite integrals and subtracting the two functions, and the integration was generally done correctly. However, there were a number of algebraic and arithmetic errors which prevented candidates from correctly showing the desired final result.

The type of assistance and guidance teachers should provide for future candidates

Although candidates seemed prepared to work without their GDCs, there were many errors in arithmetic and algebraic manipulation. In particular, candidates need to practice working with fractions and negatives, and algebraic skills such as distribution.

Candidates generally did well in situations where the question specifically led them to a specific formula found in the information booklet. However, candidates also need to be familiar with things which are not explicitly stated in the booklet, such as trigonometric values, rules of exponents and logarithms, and integrating functions such as $y = \sin(2x - 3)$.

It would be helpful for candidates to practice working under exam-type conditions. Many candidates seem to have spent too much time on some questions, while not leaving themselves enough time to work on other questions. In addition, there were a large number of candidates who seemed unfamiliar with command terms such as HENCE and SHOW THAT, and did not understand the requirements of these types of questions.

Finally, teachers should stress to their students the importance of presenting their working in an organized manner. It was noted by examiners that the strongest candidates tended to do this very well. In addition, it is best not to use graph paper for anything other than graphs, as these papers are especially difficult to read when scanned. Candidates should use lined paper for most, if not all, of their work in Section B.

Paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 – 13	14 – 27	28 – 39	40 – 49	50 – 60	61 – 70	71 – 90

The areas of the programme and examination that appeared difficult for the candidates

On this paper, a large number of candidates demonstrated a comprehensive knowledge and understanding of the syllabus. There remain areas of concern for both students and teachers. The following areas proved difficult for candidates:

- Obtaining relevant statistical values from a GDC
- Graphical solutions of equations
- Solution of a system of linear equations on a GDC
- Relationships between f, f' and f''
- Use of a trigonometric model
- Giving precise explanations for mathematical situations

The levels of knowledge, understanding and skill demonstrated

The candidates in this session generally showed a comprehensive knowledge and understanding of the syllabus. While some questions were answered better than others, fewer questions were left blank indicating that the examination was accessible to most students. Many of the problems on this paper could be traced back to either not reading the question carefully or making inappropriate or no use of a GDC when they should have.

Candidates demonstrated a great deal of skill with sequences, scalar product, cosine rule and calculus. Surprisingly, many candidates could set up and solve trigonometric equations analytically even when a graphical solution from the GDC was preferred and expected.

The area that caused the most trouble for candidates was those questions requiring the use of a GDC. Candidates for the most part do not know how to use the statistical features of their calculators or how to find the solution to a resulting system of equations.

Although there seemed to be some improvement with standard GDC operations such as graphing functions within a given domain, finding areas under or between curves or volumes of revolutions, there were still a great many candidates who have yet to master these skills. There are still many candidates who do not consider whether their calculator is in degree or radian mode.

There are candidates who are not familiar with the command terms or are not familiar with some basic mathematical terminology. For example, the "write down" command is still misunderstood and terms such as “expression” or “obtuse” seemed to be unfamiliar to a great many candidates.

Reasoning skills demonstrated by candidates were generally weak and many candidates often could not support their statements with rigorous mathematical logic.

Those candidates demonstrating skill in using both analytical and geometrical techniques had little difficulty with this paper.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Surprisingly, this question was not answered well primarily due to incorrect GDC use and a lack of understanding of the terms “median” and “interquartile range”. Many candidates opted for an analytical approach in part (a) which always resulted in mistakes. Some candidates wrote the down the mean instead of the median in part (b).

Question 2

The graph in part (a) was well done. It was pleasing to see many candidates considering the domain as they sketched their graph. Part (b) (i) asked for an expression which bewildered a great many candidates. However, few had difficulty obtaining the correct answer in (b) (ii).

Question 3

The majority of candidates could either recognize the common difference in the formula for the n^{th} term or could find it by writing out the first few terms of the sequence. Part (b) demonstrated that candidates were not familiar with expression, “n^{th} term”. Many stated that the first term was 5 and then decided to use their own version of the n^{th} term formula leading to a great many errors in (b) (ii). Some candidates managed to obtain follow through on this part.

Question 4

For the most part, this question was well done and candidates had little difficulty finding the scalar product, the appropriate magnitudes and then correctly substituting into the formula for the angle between vectors. However, few candidates were able to solve the resulting equation using their GDCs to obtain the correct answer. Problems arose when candidates attempted to solve the resulting equation analytically.

Question 5

A number of candidates left this question blank. Those that attempted it often obtained full marks. Errors were made in not simplifying the equations in part (a) or using an analytical approach rather than reverting directly to their GDC when solving the resulting system in part (b).

Question 6

Part (a) was well done with the majority of candidates obtaining the acute angle. Unfortunately, the question asked for the obtuse angle which was clearly stated and shown in the diagram. No matter which angle was used, most candidates were able to obtain full marks in part (b) with a simple application of the cosine rule.

Question 7

There were mixed results in part (a). Students were required to understand the relationships between a function and its derivative and often obtained the correct solutions with incorrect or missing reasoning. In part (b), the question was worth two marks and candidates were required to make two valid points in their explanation. There were many approaches to take here and candidates often confused their reasoning or just kept writing hoping that somewhere along the way they would say something correct to pick up the points. Many confused f' and g'.

Question 8

This question was well done by many candidates. If there were problems, it was often with incorrect or inappropriate GDC use. For example, some candidates used the trace feature to answer parts (a) and (b), which at best, only provides an approximation. Most candidates were able to set up correct expressions for parts (c) and (d) and if they had used their calculators, could find the correct answers. Some candidates omitted the important parts of the volume formula. Analytical approaches to (c) and (d) were always futile and no marks were gained.

Question 9

There was wide spectrum of success on this problem. Candidates could normally find $\mathrm{E}(X)$ using $n \times p$ but many failed to recognize that the “experiment” was binomial or that for Mark to the pass the test, he needed to answer either 3, 4 or 5 questions correctly. Part (b) was generally well done although there were a number of algebraic errors particularly in part (b) (ii), leading to incorrect values of *a* and *b*. Again, appropriate use of the GDC here would have eliminated these errors. In (c), candidates had trouble with the command term, “find” and often just wrote down either “Mark” or “Bill”.

Question 10

Part (a) was well done with most candidates obtaining the correct answer.

Part (b) however was problematic with most errors resulting from incorrect, missing or poorly drawn diagrams. Many did not recognize this as a triangle trigonometric problem while others used the law of cosines to find the chord length rather than the vertical height, but this was only valid if they then used this to complete the problem. Many candidates misinterpreted the question as one that was testing arc length and area of a sector and made little to no progress in part (b).

Still, others recognized that 6 minutes represented 1/5 of a rotation, but the majority then thought the height after 6 minutes should be 1/5 of the maximum height, treating the situation as linear. There were even a few candidates who used information given later in the question to answer part (b). Full marks are not usually awarded for this approach.

Surprisingly, part (c) was not well done. It was expected that candidates simply use the formula $\frac{2\pi}{\text{period}}$ to find the value of *b* and then substitute back into the equation to find the value of *c*. However, candidates often preferred to set up a pair of equations and attempt to solve them analytically, some successful, some not. No attempts were made to solve this system on the GDC indicating that candidates do not get exposed to many "systems" that are not linear. Confusing radians and degrees here did nothing to improve the lack of success.

In part (d), candidates were clear on what was required and set their equation equal to 96. Yet again however, solving this equation graphically using a GDC proved too daunting a task for most.

Recommendations and guidance for the teaching of future candidates

Since the inception of the GDC paper in May 2008, it has been clearly evident which centres are teaching their students both analytical and technological approaches to facilitate understanding of concepts. Many teachers are either using one or the other but do not emphasize the relationship between the two. A graphical understanding in conjunction with analytical techniques can lead to a much greater depth of understanding. As such, candidates will be able to apply their knowledge to problems presented in a slightly different way.

Candidates should be encouraged to draw additional diagrams on the answer sheets. Many showed work on the actual exam scripts, especially drawing directly on the diagram in Q10. This made it hard to follow their written work as examiners are not obliged to read what is written on the question paper!

Candidates should be encouraged to use plenty of space to communicate their work and draw figures where appropriate. It is clear that weaker candidates fail to understand the language of mathematics and are unable to communicate their understanding effectively. Stronger candidates have work that is presented clearly and concisely.

Candidates should be encouraged to use the correct features of the GDC to find maxima, minima and/or roots.

One feature of candidate performance was how often candidates reached for a formula instead of thinking through the requirements of a question. Formulas can be helpful when a calculation is required, but for questions that assess conceptual understanding, the formula approach often led candidates away from the goals of the problem.

Teachers should continue to stress the meaning of the command terms and have students look at the number of marks allocated to each question part to determine how much "work" they should show.

A fully correct expression for the area under a curve or the volume includes the limits, correct substitution of the function into the integrand and the "dx". The teacher has the responsibility to instruct candidates appropriately as to the meaning of common terms used on examinations.

Teachers need to raise awareness of need for clarity and rigour in "show that" and "explain" questions.

Teachers need to make students aware of which standard deviation to use in SL. As all data are treated as populations, candidates should be using the population standard deviation, σ .